The Laplace Transform

$$F(s) = \mathcal{L}\{f(t)\} = \int_0^\infty e^{-st} f(t)\, dt$$

$f(t)$	$F(s)$	$f(t)$	$F(s)$
1	$\frac{1}{s}$	e^{at}	$\frac{1}{s-a}$
t	$\frac{1}{s^2}$	$\cos kt$	$\frac{s}{s^2+k^2}$
t^n $(n \geq 0)$	$\frac{n!}{s^{n+1}}$	$\sin kt$	$\frac{k}{s^2+k^2}$
t^α $(\alpha > -1)$	$\frac{\Gamma(\alpha+1)}{s^{\alpha+1}}$	$\cosh kt$	$\frac{s}{s^2-k^2}$

$$\mathcal{L}\{e^{at} f(t)\} = F(s-a)$$

$f(t)$	$F(s)$	$f(t)$	$F(s)$
$e^{at} t^n$	$\frac{n!}{(s-a)^{n+1}}$	$e^{at}\cos kt$	$\frac{s-a}{(s-a)^2+k^2}$
$e^{at} t^\alpha$ $(\alpha > -1)$	$\frac{\Gamma(\alpha+1)}{(s-a)^{\alpha+1}}$	$e^{at}\sin kt$	$\frac{k}{(s-a)^2+k}$

$\mathcal{L}\{t^n f(t)\} = (-1)^n \frac{d^n F(s)}{ds^n}$ $(n \geq 1)$ $\quad \mathcal{L}\{\frac{1}{t} f(t)\} = \int_s^\infty F(t)\,dt$

$\mathcal{L}\{\int_0^t f(u)\,du\} = \frac{1}{s} F(s)$ $\quad \mathcal{L}\{\frac{\sin t}{t}\} = \frac{\pi}{2} - \arctan s$

$u(t) = \begin{cases} 0, & t < 0 \\ 1, & t \geq 0 \end{cases}$ $\quad \mathcal{L}\{u(t)\} = \frac{1}{s}$

$u_a(t) = u(t-a) = \begin{cases} 0, & t < a \\ 1, & t \geq a \end{cases}$ $\quad \mathcal{L}\{u_a(t)\} = \frac{e^{-at}}{s}$

$\mathcal{L}\{f'(t)\} = s\,\mathcal{L}\{f(t)\} - f(0) = sF(0) - f(0)$

$\mathcal{L}\{f''(t)\} = s^2 F(s) - sf(0) - f'(0)$

$f(t) * g(t) = \int_0^t f(u) g(t-u)\,du$, $\quad \mathcal{L}\{f(t) * g(t)\} = \mathcal{L}\{f(t)\} \cdot \mathcal{L}\{g(t)\} = F(s) \cdot G(s)$

$f(t) = \mathcal{L}^{-1}\{F(s)\}$, $\quad \mathcal{L}^{-1}\{aF(s) + bG(s)\} = af(t) + bg(t)$

$\mathcal{L}^{-1}(1) = \delta(t) = \begin{cases} +\infty, & t = 0 \\ 0, & t \neq 0 \end{cases}$

$\delta_a(t) = \begin{cases} +\infty & \text{if } t = a \\ 0 & \text{if } t \neq a \end{cases}$, $\quad \int_0^\infty \delta_a(t) g(t)\,dt = g(a)$