

Number theory-->Modular arithmetic-->modular congruence 【模同余基本知识要点】

1) If a is congruent to b modulo m , then you write $a \equiv b \pmod{m}$. that is that a and b leave the same remainder when they are divided by m . 【 a, b 对模 m 同余, 记作 $a \equiv b \pmod{m}$, 就是 a, b 被 m 除余数相同】.

2) Let m be a positive integer. If a and b are integers, then a is congruent to b modulo m if $m | (a-b)$. 【 a 和 b 是整数, m 是正整数, 如果 $m | (a-b)$, 那么, $a \equiv b \pmod{m}$ 】

3) If $a, b \in \mathbb{Z}$, then $a \equiv b \pmod{m}$ for some positive integer m if and only if there exists an integer k such that $a = b + km$. 【 a, b 是整数, m 是正整数, $a \equiv b \pmod{m} \iff a = b + km$ 】

4) If $a \equiv b \pmod{m}$ and $b \equiv c \pmod{m}$, then $a \equiv c \pmod{m}$ 【同余的传递性】

5) Let $a, b, c, d, m, n \in \mathbb{Z}$ and $m, n > 0$, with $a \equiv b \pmod{m}$ and $c \equiv d \pmod{m}$. then:

$$a \pm c \equiv b \pm d \pmod{m}$$

$$ac \equiv bd \pmod{m}$$

$$ka \equiv kb \pmod{m}$$

$$a^n \equiv b^n \pmod{m}$$

6) Fermat's little theorem states that, if p is a prime number and a is any number not divisible by p , then $a^{p-1} \equiv 1 \pmod{p}$ and $a^p \equiv a \pmod{p}$. 【费马小理: 如果 p 是质数, $p \nmid a$, 那么, $a^{p-1} \equiv 1 \pmod{p}$ 和 $a^p \equiv a \pmod{p}$ 】

7) A multiplicative inverse of a modulo m is an integer p that satisfies $ap \equiv 1 \pmod{m}$ the multiplicative inverse exists if and only if $\gcd(a, m) = 1$ 【如果 $\gcd(a, m) = 1$ 和 $ap \equiv 1 \pmod{m}$, p 是 a 的同余乘逆】

8) If a and b are positive integers and p is a prime number with $p \nmid a$, then a^{p-2} is a multiplicative inverse of a modulo p and the solution to the equation $ax \equiv b \pmod{p}$ is given by $x \equiv a^{p-2}b \pmod{p}$ 【 p 是质数, $p \nmid a$, a^{p-2} 是 a 的同余乘逆, $ax \equiv b \pmod{p}$ 解是 $x \equiv a^{p-2}b \pmod{p}$ 】

9) Let $a, b, m \in \mathbb{Z}$, with $m > 0$ and $\gcd(a, m) = d$

If $d \nmid b$ then the equation $ax \equiv b \pmod{m}$ has no solutions.

If $d \mid b$ then the equation $ax \equiv b \pmod{m}$ has d solutions in the set of least residues modulo m . 【如果 $d \mid b$, 则方程 $ax \equiv b \pmod{m}$ 在最小模剩余集中有 d 个解】

10) If $ka \equiv kb \pmod{m}$ and $\gcd(k, m) = 1$ then $a \equiv b \pmod{m}$

If $ka \equiv kb \pmod{m}$ and $\gcd(k, m) = d$ then $a \equiv b \pmod{m/d}$